EFFECT OF THE PULSATIONS OF A LIQUID ON THE AVERAGED MOTION OF A SUSPENDED PARTICLE

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It is shown that particles moving in a pulsating flow with relative velocities extending outside the normal Stokes range acquire average velocities differing from those normally associated with sedimentation.

Many technological processes involve the motion of dust or silt in a pulsating flow of liquid. In the usual methods employed for calculating the velocities and trajectories of the particles, it is tacitly assumed that the alternating effects of the accelerating and retarding pulsations on the motion of a particle compensate each other, and the average motion remains the same as in the absence of pulsations.

In order to verify the validity of this assumption, let us consider the effect of the pulsations of a liquid on the averaged motion of a suspended particle in general form, confining attention to the solution of the one-dimensional problem. Let us assume that periodic pulsations of arbitrary form are superimposed on the uniform averaged motion of the flow, while the acceleration of the external mass force acting on the particle is constant.

If we neglect the influence of the augmented mass and the effect of acceleration on the resistance coefficient, the particle will be acted upon by the resistive forces and by the external and generalized Archimedes force ($-\pi\delta^3/6$ grad p), which for an ideal liquid will be equal to $m_1(j-dv/dt)$. Then the equation of motion of a spherical particle takes the form

$$m_2\frac{dw}{dt}=-k\frac{\pi\delta^2}{4}\cdot\frac{\rho_1 u|u|}{2}+(m_2-m_1)j+m_1\frac{dv}{dt}. \tag{1}$$

Substituting $w = v + u$, $t = \tau T$, $v = v_a V$ and using the similarity criteria Re, Ar, Stk, and R, we reduce (1) to the dimensionless form

$$\frac{d\,\mathrm{Re}}{d\tau}=-R\frac{dV}{d\tau}-\frac{1}{\mathrm{Stk}}(\psi\,\mathrm{Re}-\mathrm{Ar}), \tag{2}$$

where Re is the dimensionless relative velocity of the particle and $V(\tau)$ is the dimensionless velocity of the pulsations of the liquid, with a period equal to unity.

Analyzing the motion of the particle in the pulsating medium, we readily see that $\mathrm{Re}(\tau)$ is a periodic function, not depending on the initial velocity, and synchronous with $V(\tau)$.

Let us consider the steady periodic motion of the particle. In this case Re and $\psi\,\mathrm{Re} = A$ may be expressed in the form $\mathrm{Re}(\tau) = \mathrm{Re}_0 + \widetilde{\mathrm{Re}}(\tau)$, $A = A_0 + \tilde{A}(\tau)$, where Re_0 and A_0 are the constant and $\widetilde{\mathrm{Re}}(\tau)$ and $\tilde{A}(\tau)$ the variable (periodic) components of $\mathrm{Re}(\tau)$ and $A(\tau)$. According to definition

$$A_0=\int_0^1 A(\tau)\,d\tau=\int_0^1 \psi\,\mathrm{Re}\,d\tau.$$

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Putting $\psi = \psi_0 + \Delta\psi$, where ψ_0 is the value of ψ for $Re = Re_0$, we have

$$A_0 = \psi_0 Re_0 + \int_0^1 \Delta\psi \, Re \, d\tau. \tag{3}$$

Let us now find A_0 from (2):

$$A_0 = -R \cdot Stk \int_0^1 dV(\tau) - Stk \int_0^1 d\,Re(\tau) + Ar \int_0^1 d\tau.$$

Since $V(\tau)$ and $Re(\tau)$ are periodic functions, we have $A_0 = Ar$.

For the uniform motion of a particle in a liquid moving at a constant velocity, i.e., for $Re = Re_S = \text{const}$, Equation (2) leads to $Ar = \psi_S Re_S$, and from (3) we obtain

$$\psi_0 Re_0 = \psi_s Re_s + C, \tag{4}$$

where

$$C = -\int_0^1 \Delta\psi \, Re \, d\tau. \tag{5}$$

The quantity Ar may be considered as a dimensionless external force acting on the particle. Hence on the basis of Eq. (4) the quantity C may also be considered as an additional force arising under the influence of the velocity pulsations. In the presence of this component, which we may call the pulsational force, Equation (4) leads to the relations $\psi_0 Re_0 \neq \psi_S Re_S$ or $Re_0 \neq Re_S$.

Let us consider several cases in which the presence or absence of a pulsational force (and also the sign of the latter) may be established without solving the nonlinear Eq. (2) and the integral (5). For this purpose let us determine the form of the function $\Delta\psi Re = (\psi - \psi_0)Re$. It may easily be shown that, since ψ is a unique function of $|Re|$, $\Delta\psi Re$ is an odd function of Re.

In the range of applicability of the Stokes formula ($|Re| < Re_C$) $\psi = 24$ and $\Delta\psi Re = (24 - \psi_0)Re$. If $|Re_0| < Re_C$, then $\psi = 24$ and $\Delta\psi Re = 0$ (the curve of $\Delta\psi Re$ coincides with the horizontal axis in the range Re_C-Re_C). However, if $Re_0 > Re_C$, then $\psi_0 > 24$, and $\Delta\psi Re$ is represented by an inclined curve passing through the origin of coordinates (Fig. 1).

In order to find the curve of $\Delta\psi Re$ outside this range, we considered the derivatives of $\Delta\psi Re$ with respect to Re:

$$(\Delta\psi\, Re)' = (\Delta\psi)'\, Re + \Delta\psi = \psi'\, Re + \psi - \psi_0,$$

$$(\Delta\psi\, Re)'' = 2\psi' + \psi''\, Re.$$

The quantities $2\psi' + \psi'' Re$ were determined graphically and by numerical differentiation of the $\psi = k|Re|$ curve, where $k = f|Re|$ is a certain relationship derived from the experimental investigations of a large number of authors, which we smoothed by the method of least squares to an accuracy of one unit in the fourth place of decimals. For $Re_C < Re < 10^5$ $(\Delta\psi Re)''$ is a finite positive quantity. Hence in this region the $\Delta\psi Re$ curve is concave and in the range $10^5 \leq Re \leq -Re_C$ convex (curve I–I in Fig. 1).

Making use of the foregoing conclusions regarding the shape of the $\Delta\psi Re$ curve, let us now analyze Eq. (5).

I. For $-Re_C < Re < Re_C$ the quantity $\Delta\psi Re = 0$, and hence, on the basis of Eq. (5), $C \equiv 0$ for any form of the function $Re(\tau)$. Hence for a linear law of resistance the pulsations of the liquid have no effect on the average motion of the particle.

Now let us consider the motion of a particle in the case of a nonlinear law of resistance, when the value of Re passes outside the range $-Re_C$ to Re_C and $Re_0 > 0$ (the case $Re_0 < 0$ cannot be considered, as this ab initio supposes the existence of a negative force $C < -Ar$).

II. Let us write down the equation of the tangent 1–1 to the curve $\Delta\psi Re$ at the point $Re = Re_0$:

$$\Delta\psi\, Re = (\Delta\psi\, Re)'_0 (Re - Re_0) = \psi'_0 Re_0 (Re - Re_0). \tag{6}$$

The tangent 1–1 intersects the curve I–I only at a point $Re_1 < -Re_0$ (Fig. 1), and hence the curve of $\Delta\psi Re$ lies above the tangent for $Re > Re_1$, whence the integral (5) is

$$\int_0^1 \Delta\psi \, Re \, d\tau > \psi_0' \, Re_0 \int_0^1 \widetilde{Re} \, d\tau.$$

Here ψ_0' and Re_0 are finite quantities and the second integral is zero, so that the first integral is positive and the quantity $C < 0$.

III. $Re_0 < Re_c$, $Re_{max} > Re_c$, $|Re_{max}| > |R_{min}|$. In this case part of the $\Delta\psi Re$ curve with $Re > Re_c$ also lies above the tangent (horizontal axis) and in the range $-Re_c < Re < Re_c$ coincides with it. Hence in this case also $C < 0$.

In the two latter cases, depending on the relation between Re_{min}, Re_0, and Re_{max}, the positive and negative amplitudes of the $\widetilde{Re}(\tau)$ curve may be either the same or different, and the $\widetilde{Re}(\tau)$ curve itself may be either symmetrical or asymmetrical.

IV. If $Re_0 < Re_c$, but $Re_{max} < Re_c$ and $|Re_{min}| > |Re_{max}|$, the $\Delta\psi Re$ curve will lie below the tangent (horizontal axis) for $Re < -Re_c$, and on the basis of the earlier argument $C > 0$. In this case the negative amplitudes of $\widetilde{Re}$ and Re are always greater than the positive, and the curve has a markedly asymmetrical character.

V. $\widetilde{Re}(\tau)$ is an odd function: $\widetilde{Re}(-\tau) = -\widetilde{Re}(\tau)$ or corresponds to the condition: $\widetilde{Re}(0.5-\tau) = -\widetilde{Re}(-\tau)$.

Using the oddness of the function $\Delta\psi Re$, we transform (5) by means of the well-known identities $\int_{-a}^{0} f(x)dx = \int_0^a f(-x)dx$ and $\int_0^a f(x)dx = \int_0^a f(a-x)dx$. For the two cases we then have

$$C = -\int_{-0.5}^{+0.5} \Delta\psi \, Re \, d\tau = -\int_0^{0,5} \{\Delta\psi \, Re \, [Re(\tau)] - \Delta\psi \, Re \, [Re(\tau) - 2 \, Re_0]\} \, d\tau.$$

In the latter integral the second integrand function is represented by the curve II–II (Fig. 1), shifted equidistantly to the right by an amount $2Re_0$ with respect to curve I–I. It follows from a comparison of the curves that the integrand is always positive, and in contrast to case II we have $C < 0$ for any values of Re.

For $Re_0 < 0$, the curve II–II would be shifted to the left with respect to curve I–I; hence the latter integral would be negative and $C > 0$; however, this is impossible, since for a positive C we should have $Re_0 > Re_s > 0$. Hence for symmetrical pulsations $-Ar < C < 0$, and a positive pulsation force is quite impossible, so that the averaged velocity of the particle cannot be raised above the sedimentation velocity, nor can it move against the external force.

Let us also consider the theoretically interesting question of motion in the absence of an external force (this may be realized, for example, if the force of gravity is balanced by an electrostatic force), when $Ar = 0$, $Re_s = 0$. Then in case IV (asymmetrical fluctuations) a positive pulsation force will produce an averaged motion of the particle at a velocity given by Eq. (4), which will here take the form $\psi_0 Re_0 = C$. In the case of symmetrical pulsations (V), however, on the basis of the inequality $-Ar < C < 0$ we have $C \equiv 0$. Thus, in the absence of an external force, the creation of an average motion of the particle relative to the liquid solely as a result of the pulsations of the latter is only possible for the case of asymmetrical fluctuations. The direction of this motion is opposite to the maximum amplitude of the relative velocity of the particle.

On the basis of the foregoing analysis, we may assert that, in many cases of the motion of fine particles in a pulsating flow, the actual average relative velocity of a particle may be expected to differ from its sedimentation velocity; the actual value may be either greater than (case IV) or less than (cases II, III, V) the latter.

The value of the pulsation force C cannot be estimated in general form, since the nonlinear relation (2) is insoluble in quadratures. In order to indicate the order of magnitude of this quantity, we therefore made some calculations for the simplest particular case in which $Re(\tau)$ was described by a rectangular graph: $Re = Re_1$ for $0 < \tau < \tau_1$, $Re = Re_2$ for $\tau_1 < \tau < 1$ (Fig. 2a). Equation (2) is then reduced from differential to algebraical form

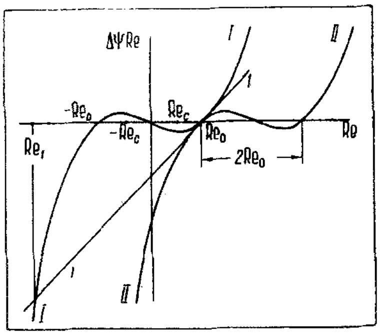

Fig. 1. Dependence of the function $\Delta\psi\mathrm{Re}$ on Re for $\mathrm{Re}_0 > \mathrm{Re}_c$: I–I curve of $\Delta\psi\mathrm{Re(Re)}$; II–II auxiliary function $\Delta\psi\mathrm{Re}(\mathrm{Re}-2\mathrm{Re}_0)$; 1–1 tangent to the curve I–I at the point Re_0.

$$\psi\,\mathrm{Re} = -NV' + \mathrm{Ar}. \tag{7}$$

Since in each section $\psi\mathrm{Re} = \mathrm{const}$, we correspondingly have $V' = \mathrm{const}$ (the graph of $V'(\tau)$ is also rectangular).

So that the value of V averaged over the period is equal to zero, we take $V_1' = 1-\tau_1$ and $V_2' = -\tau_1$. Then for each part Eq. 7 takes the form

$$\begin{gathered}\psi_1\,\mathrm{Re}_1 = -N(1-\tau_1) + \mathrm{Ar},\\ \psi_2\,\mathrm{Re}_2 = N\tau_1 + \mathrm{Ar}.\end{gathered} \tag{8}$$

Since the absolute velocity of the particle cannot be a discontinuous function of time, we have $w(\tau_1 - 0) = w(\tau_1 + 0)$ or $RV_1 + \mathrm{Re}_1 = RV_2 + \mathrm{Re}_2$, whence $V_1 - V_2 = (\mathrm{Re}_2 - \mathrm{Re}_1)/R$.

Hence $V(\tau)$ has breaks in continuity at the boundaries of the sections (Fig. 2b), while $W(\tau)$ is represented by a saw-toothed curve (Fig. 2c).

Using Eq. (8), we made a number of calculations for symmetrical ($\tau_1 = 0.5$) and asymmetrical pulsations of Re for $10^{-2} \le N \le 10^7$ and $10^{-3} \le \mathrm{Ar} \le 10^4$. These values of the criteria may be realized for the motion of particles with a density of $\rho_2 = 2000\ \mathrm{kg/m^3}$ in air at 20°C and values of δ, j and v_a/T within the ranges 1-500 μ, 9.81-10^4 m/sec², and 10^{-4}-10^3 m/sec². The maximum value of $j = 10^4$ may be achieved, for example, in a dust catcher of diameter 200 mm for an inlet velocity of 30 m/sec. The range of v_a/T indicated may be achieved under conditions of turbulent pulsations when the velocity v and the tube radius r vary over the ranges 1-50 m/sec and 0.01-3 m. If pulsations are applied artifically, much greater values of N may be achieved than in turbulent flows. In a sonic field, for example, values of $v_a/T = 10^3$ m/sec² may be obtained for sound intensities of 100-130 dB and frequencies of 1-10 kHz.

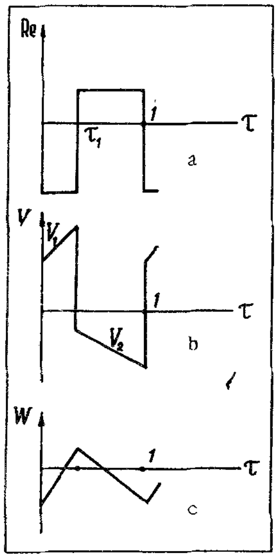

Fig. 2. Dependence of the relative velocity of the particles Re (a), the velocity of the flow V (b), and the absolute velocity of the particle W (c) on time τ.

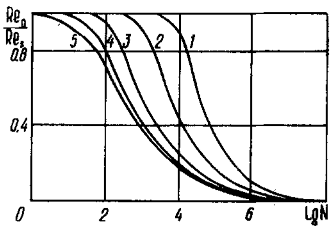

Fig. 3

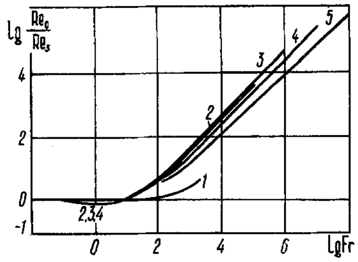

Fig. 4

Fig. 3. Averaged velocity of a particle for symmetrical rectangular pulsations of relative velocity (τ_1 = 0.5) in relation to the N and Ar criteria: 1) Ar = 10^4; 2) 10^3; 3) 10^2; 4) 10; 5) 10^{-3}.

Fig. 4. Averaged velocity of a particle for asymmetrical pulsations of relative velocity (τ_1 = 0.25) in relation to the N and Fr criteria: 1) N = 10^0; 2) 10^2; 3) 10^3; 4) 10^4; 5) 10^6.

Figure 3 illustrates the effect of the determining criteria on the motion of the particle in the form of a curve relating Re_0/Re_S to the criteria N and Ar for symmetrical pulsations of relative velocity. The velocity of the particle falls on increasing N, i.e., on increasing the action of the pulsations in the flow (for a constant value of the external force). Thus, for example, in the case of a particle 100 μ in size, its mean velocity for an acceleration of 9.81 m/sec^2 by the external force and v_a/T = 10 m/sec^2 is (according to calculation) approximately 94% of the sedimentation velocity, while, on further increasing v_a/T (the criterion N), Re_0/Re_S falls and on the limit tends to zero, i.e., $C \to -Ar$.

The foregoing theoretical conclusions, which imply the retardation of the relative motion of the particles under the influence of symmetrical pulsations, were also confirmed by solving the differential equation of motion of the particle by the Runge–Kutta method in a high-speed computer. Fluctuations in the rate of flow of a sinusoidal form (for example, with v_a = 0.5 m/sec and a frequency of 100 Hz) cause the sedimentation velocity of a particle of diameter δ = 100 μ to fall from 0.556 to 0.407 m/sec (the corresponding values of the criteria are R = 3.25; Stk = 145; Ar = 93.5; Re_0/Re_S = 0.73).

For asymmetrical pulsations, the value of the pulsation force and the average velocity of the particle depend on the type of asymmetry of the pulsations, and for $\tau_1 \geq 0.5$ the force is always negative, while for $\tau_1 < 0.5$ C may be either negative or positive.

This is because large values of ψ and of the resistive force correspond to the motion of a particle with a greater peak velocity, the direction of which coincides with or opposes the direction of action of the external force, which depends on τ_1 (Fig. 2a).

It was found that the effect of the pulsations only appeared appreciably for values of N > Ar. We therefore evaluated Re_0/Re_S for asymmetrical pulsations as a function of the criteria N and Fr = N/Ar; an example for τ_1 = 0.25 is presented in Fig. 4. All the curves in Fig. 4 pass close to the point Fr = 10 and Re_0/Re_S = 1. This shows that, up to N = 10Ar, the existence of the pulsations with τ_1 = 0.25 has no appreciable effect, except for large values of the criterion N. In our own case of N = 10^2 there is a slight retardation of the particle (C < 0) for values of $-1 < \log Fr < 1$. For values of Fr > 10 there is an increase in the particle velocity (C > 0) in the direction of the positive axis. In conclusion, we may thus present the following points:

1) for the motion of a particle in a pulsating flow of liquid at relative velocities extending outside the Stokes region, the averaged velocity of the particle may differ from the velocity in an unperturbed liquid. In this case the accelerating and retarding effects of the pulsations of the liquid on the motion of the particle do not compensate each other, and an additional "pulsation" force arises;

2) this force may be either positive or negative;

3) for any symmetrical pulsations of the relative velocity of the particle, the pulsation force is always negative, but smaller (in absolute magnitude) than the external force;

4) numerical calculations confirm the conclusions of the theoretical analysis, and show that, in the presence of symmetrical pulsations of the relative velocity of the particle, a negative pulsation force is created, leading to a reduction in the averaged particle velocity. For asymmetrical pulsations, the absolute value of the pulsation force may be several orders of magnitude greater than the external force.

NOTATION

δ, ρ_2	are the diameter, m, and density, kg/m^3, of the particle;
w, u	are the absolute velocity of particle and velocity relative to the liquid, m/sec;
v, ρ_1, η, v_a	are the velocity, m/sec, density, kg/m^3, dynamic viscosity, N·sec/m^2, and amplitude of the velocity m/sec, of the liquid;
τ	is the time, sec;
T	is the period, sec;
j	is the acceleration of the external mass force, m/sec^2;
p	is the pressure, N/m^2;
m_2 and m_1	are the mass of particle and of the liquid displaced by the latter, kg;
k and $\psi = k\lvert Re \rvert$	are resistance coefficients;

$Re = u\delta\rho_1/\eta$,
$Stk = 4\rho_2\delta^2/3\eta T$;
$R = v_a\delta\rho_1(\rho_2 - \rho_1)/\eta\rho_2$;
$Fr = v_a/jT$,
$Ar = 4j(\rho_2 - \rho_1)\delta^3\rho_1/3\eta^2$;
$N = R\,Stk$.